**AE68714 Homework #3**

**Student: Trinh Minh Hoang ID: 202383209**

Consider the following periodic function (from last homework) for the problem 3.1~3.4

**Problem 3.1.** You will run the provided MATLAB code to find the approximated first derivative of the function given on 16, 32, 64, and 128 point (M) meshes using 3-pt central differencing:

It uses periodic boundary conditions (L = 2π). Please provide nice write-up that covers the following (reference figures by number in the text):

1. Provide a short description of the problem to be examined and how the provided program implements the algorithm (How it handles the periodic boundaries and how it does not perform matrix-vector multiplications)
2. Run and include the plot of the approximated first derivative values versus for the four meshes (M=16, 32, 64, 128) on one figure and comment about the results.
3. What is the shape of the actual errors from the finest mesh and the coarsest mesh? Why might it be different (plot on the second figure)?
4. Make a table of the predicted values and errors at as a function of ∆𝑥. Is this consistent with what you got in Problem (2.5) from the last homework?
5. Plot the 𝐿1 norm of the grid error versus ∆𝑥 on a log − log plot (on the third figure). What observation can you make about the increased accuracy as the solution is refined.

**Problem 3.2.** Repeat the problem (3.1) using 4-pt backward biased differencing:

Please provide nice write-up that covers the following (reference figure by number in text):

1. Run and include the plot of the approximated first derivative values versus x for the four meshes (M = 16, 32, 64, 128), on one figure and comment about the results.
2. Plot the actual error in the approximated derivative values versus , for the four meshes, on a second figure and comment about the results. What is the shape of the errors? Why might it be different for the finest mesh as compared to the coarsest mesh ?
3. Make a table of the predicted values and errors at = π/8 as a function of ∆𝑥. What observations can you make about the increased accuracy as the solution is refined ?
4. Plot the 𝐿1 norm of the grid error versus ∆𝑥 on a log − log plot (on the third figure). What observation can you make about the increased accuracy as the solution is refined ?

**Problem 3.3.** Construct a Taylor Table for the following Hermetian (compact) scheme:

and find the resulting Taylor series error*,*  . What is the order of the method ?

**Problem 3.4.** Repeat the problem (3.1) using the following Hermetian (compact) differencing:

You will need to modify the provided program to add this method as an option. You can use a periodic tridiagonal solver (trip.m) to solve the coupled system of equations. Please provide a nice write-up that covers the following (reference figures by number in text):

1. Provide a listing of your final modified program that includes this method and describe in words the modifications.
2. Run and include the plot of the approximated first derivative values versus x for the four meshes (M = 16, 32, 64, 128), on one figure and comment about the results.
3. Plot the actual error (not 𝑙𝑜𝑔10 of the absolute error) in the approximated derivative values versus x, for the four meshes, on a second figure and comment about the results. What is the shape of the errors? Why might it be different for the finest mesh as compared to the coarsest mesh ?
4. Make a table of the predicted values and errors at 𝑥 = π/8 as a function of ∆𝑥. What observations can you make about the increased accuracy as the solution is refined ?
5. Plot the 𝐿1 norm of the grid error versus ∆𝑥 on a log − log plot (on the third figure). What observation can you make about the increased accuracy as the solution is refined ?

**Problem 3.5.** Find the expression for the modified wave number times ∆𝑥 (i.e. for the following finite difference approximations to in terms of 𝑘∆𝑥 (i.e. let):

1. 5-pt Lagrangian Power Polynomial (4th order accurate)
2. 5-pt Mixed Trigonometric/Power Polynomial (N=4 for trig. Terms) (2nd order accurate):
3. 3-pt Hermetian Power Polynomial (4th order accurate)
4. 4-pt Lagrangian Power Polynomial (3rd order accurate)
5. 3-pt Hermetian Power Polynomial (3rd order accurate)

**Problem 3.6.** For the schemes from problem (3.5) plot the resulting expressions for the against 𝑘∆𝑥 from 0 to π.

(a) For central difference schemes should be real. Please plot the real parts versus 𝑘∆𝑥 on one figure for the schemes in 3.5 (a) through (c). Also plot the exact relation: = 𝑘∆𝑥

(b) For biased schemes (**3.5** (d) and (e)) the results for should all be complex. Please plot the real parts versus 𝑘∆𝑥 in one figure (along with the exact relation: = 𝑘∆𝑥). In another figure plot the imaginary parts against 𝑘∆𝑥 (dissipation resolution - why?)

(c) Please note any observations and discuss. What is the effect of trigonometric terms ? What is the effect of Hermetian interpolation ? What is the effect of biased versus central ? etc